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Immunitization of Pension Funds and Sensitivity to Actuarial Assumptions: Comment

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A recent article by Keintz and Stickney (K-S) [5] outlines a bond duration method for immunizing pension plans from market rate risk. The authors show how unexpected alterations in market rates of return can cause offsetting changes in pension assets and liabilities. By employing Macaulay's bond duration formula, K-S demonstrate how these opposing balance sheet changes may protect a pension plan from random market rate movements. Their research should prove quite useful in managing pension funds where the investment instruments have known maturities. Some pension plans, though, may desire to invest in corporate bonds where call features could alter maturity structure unexpectedly.

K-S raise the issue of the effect corporate bonds with callable features could have on duration. The K-S technique relies to some extent on the pension manager matching the maturity structure of investment assets with fund liabilities.¹ A call provision allows the borrower to redeem bonds before maturity and thus may impose uncertainty on the length of the asset holding period. Many corporate bond issues will include call privileges which permit early redemption.² Ostensibly the purpose of the call feature is to provide the corporation with the chance to refinance their bonds when interest rates are lower than when the bonds were originally issued. A bond call provision subjects the investor to the risk of redemption at a time of low interest, thus reducing overall investment yield and uncertainty as to when bond proceeds are payable. Most investors protect against this type of interest rate risk by requiring a higher yield on callable issues. However, the uncertainty with respect to maturity structure of callable issues is one over which the investor has little control.

Figure 1 depicts the type of situation faced by the investor in callable securities. Condition A prevails as long as market interest rates during the call period are above the effective yield on the bond. Condition B necessitates

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¹ The same hedging technique of matching asset and liability holding periods has been used by banks, savings and loan associations and other financial institutions with varying degrees of success, see [2].

² For a discussion of corporate call provisions with the advantages and disadvantages for borrower and investors alike, see [9].

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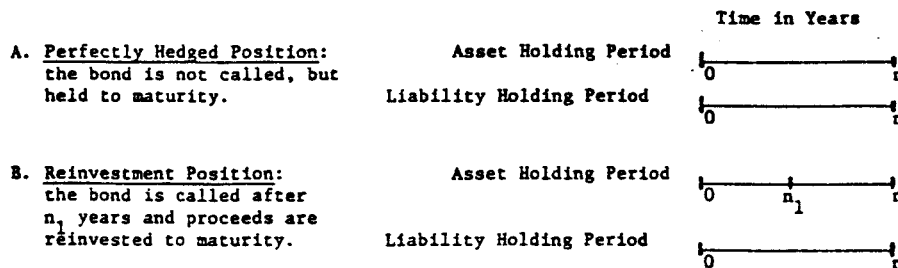
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Figure 1

Asset and Liability Holding Periods for Callable Bonds



reinvesting bond proceeds at a time when market rates of interest are below the original bond yield rate.³ The duration principle utilizes the notion that as market rates of return decrease, discount rates decrease, and the present value of pension liabilities and fund assets will increase by an offsetting amount.⁴ However, under condition B the bond is called for a fixed redemption value which may prevent fund assets from growing to meet the higher liabilities caused from lower market rates of return.]

It is possible to examine the effect of callable features on duration by considering an alternative formulation for asset duration. Duration for a bond without a call provision can be defined as:

$$D_t^A = \frac{(Fr) (Ia)_{\overline{t}|i} + Cv_i^t}{TA} \quad (1)$$

- where:
- F = par value on the bond
 - r = coupon rate on the bond
 - t = number of interest conversion periods to maturity
 - C = maturity value on the bond
 - i = discount rate per interest conversion period,
 - TA = $(Fr) a_{\overline{t}|i} + Cv_i^t + CB$, which are exogeneously defined total assets, with CB = cash balances over the holding period,
 - $(Ia)_{\overline{t}|i}$, $a_{\overline{t}|i}$ and v_i^t are the usual interest rate functions.⁵

³ To some extent condition B appears to be a bond replacement problem which is investigated in [3] and [6]. However, condition B results from the borrower's decision to call an issue, rather than the investor's choice of replacing the bond. Fisher and Weil mention corporate callable bonds in their paper, but exclude doing any analysis on the problem. See [6], pp. 414-15.

⁴ If market rates of return increase over the asset holding period, then condition A appears and a fortiori, fund assets and liabilities decrease with increases in the discount rate, so that the benefits of duration apply.

⁵ This formula is similar to the one used in [5] except that the bond here has coupon interest separate from repayment of principal. In addition, the formula in [5] at the top of page 225 probably should have indices which start at zero in both numerator and demoninator.

Relationships for asset duration similar to the empirical results for liability duration given by K-S can be demonstrated by differentiating (1).

$$\frac{d}{dj} D_t^A = \frac{-1}{TA} \{ (Fr)_{k=0}^{t-1} [(k+1)^2 v_1^{k+2}] + tCv_1^{t+1} \} < 0 \quad (2)$$

$$\frac{d}{dt} D_t^A = \frac{v_1^t}{TA} \{ (Fr) \left[\frac{\delta(1+i)}{i^2} - \frac{-(1-t\delta)}{i} \right] - C\delta \} > 0 \quad (3)$$

where: $\delta = \log_e(1+i)$.⁶

Differentiation (2) implies that duration is inversely related to the rate of discount, whereas (3) indicates a direct relation between duration and the length of the asset holding period. Exhibit 4 in K-S shows that liability duration is indirectly related to discount rates and employee turnover. Since employee turnover shortens the holding period of liabilities, a positive relationship exists between duration and liability holding periods. Thus changes in the discount rate or the asset holding period have an impact on duration analogous to the K-S results for liabilities.

Duration for a bond where the call provision is exercised can be defined as:

$$D_t^A = \frac{(Fr)(Ia)_{\overline{n}|i} + C(v_1^n)(v_1^{t-n}) + (Fr)(v_1^n) \left[na_{\overline{t-n}|j} + (Ia)_{\overline{t-n}|j} \right]}{TA} \quad (4)$$

where: F = par value on the bond,

r = coupon rate,

C = redemption value,

i = n-period interest rate,

j = interest rate on a new bond after the call,

n = the number of interest conversion periods before the call,

t-n = the number of interest conversion periods after the call,

TA = total assets held throughout the holding period which is exogenously given,

$a_{\overline{t-n}|j}$, $(Ia)_{\overline{n}|i}$, v_1^n , v_1^{t-n} , $(Ia)_{\overline{t-n}|j}$, are the usual interest rate functions.

Formula (4) is based on the assumption that proceeds after the call are reinvested in a bond similar to the original, offering coupon income (Fr) with maturity in t-n periods at the prevailing yield rate j. Since the call occurs in the nth period, then j is less than i during the t-n periods to the end of the holding period.

⁶ Since $\frac{\{(Fr)\delta(1+i)\}}{i^2} > C\delta$ and $\frac{\{t \log_e(1+i)\}}{i} > \frac{1}{i}$ for t different from zero, given that $\log_e(1+i) = i - 1/2 i^2 + 1/3 i^3 - \dots$

Differentiating (1) relationships:

$$\frac{d}{di} D_t^A = \frac{-1}{TA} \{ (Fr)_{k=0}^{t-1} \}$$

$$\frac{d}{di} D_t^A < 0$$

$$\frac{d}{dj} D_t^A = \frac{-1}{TA} \{ v_1^n (t-n) \}$$

$$\frac{d}{dj} D_t^A < 0$$

$$\frac{d}{dn} D_t^A = \frac{(Fr)}{TA}$$

$$+ Cv_1^n$$

$$+ \frac{(Fr)}{TA}$$

$$+ \frac{(Fr)}{TA}$$

$$\frac{d}{dn} D_t^A > 0$$

where $\delta = \log_e$

According to (5) and reinvestment rate j ap case. Equation (7) in bond purchase date. C by allowing n to equal the reinvestment rate callable security loss maturity bond.

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⁷ Using an argume $(Fr) \left\{ \frac{1}{i^2} \delta(1+i) + \frac{\delta'}{j^2} \right\}$

$$\frac{1}{i} (n\delta) > \frac{1}{i}, \text{ and } \frac{1}{j}$$

Differentiating (4) with respect to i , j , and n establishes the following relationships:

$$\frac{d}{di} D_t^{A'} = \frac{-1}{TA} \{ (Fr) \sum_{k=0}^{n-1} \{ (k+1)^2 v_i^{k+2} \} + n v_i^{n+1} [C v_j^{t-n} + (Fr) (n a_{\overline{t-n}|j} + (Ia)_{\overline{t-n}|j})] \}$$

$$\frac{d}{di} D_t^{A'} < 0 \tag{5}$$

$$\frac{d}{dj} D_t^{A'} = \frac{-1}{TA} \{ v_i^n (t-n) v_j^{t-n+1} + (Fr) (v_i^n) [n v_j (Ia)_{\overline{t-n}|j} + \sum_{k=0}^{t-n-1} (k+1)^2 v_j^{k+2}] \}$$

$$\frac{d}{dj} D_t^{A'} < 0 \tag{6}$$

$$\begin{aligned} \frac{d}{dn} D_t^{A'} = & \frac{(Fr)}{TA} \left\{ \frac{1}{i} \left[\frac{\delta(1+i)}{i} v_i^n - v_i^n + n v_i^n \right] \right\} + C v_j^{t-n} (v_i^n) (-\delta) \\ & + C v_i^n (v_j^{t-n}) (-\delta') + \frac{(Fr)}{TA} \{ -v_i^n [n a_{\overline{t-n}|j} + (Ia)_{\overline{t-n}|j}] \\ & + \frac{(Fr)}{TA} \{ v_i^n a_{\overline{t-n}|j} + \frac{n \delta'}{j} v_j^{t-n} \} \\ & + \frac{(Fr)}{TA} \left\{ \frac{v_i^n}{j} \left[\frac{\delta'(1+j)}{j} v_j^{t-n} - v_j^{t-n} + (t-n) v_j^{t-n} \delta' \right] \right\} \end{aligned} \tag{7}$$

$$\frac{d}{dn} D_t^{A'} > 0$$

where $\delta = \log_e(1+i)$ and $\delta' = \log_e(1+j)$.⁷

According to (5) and (6) duration will increase with decreases in i and j . As the reinvestment rate j approaches i , equations (5) and (6) approach (2) as a polar case. Equation (7) implies that the longer the call date is from the original bond purchase date, duration increases. Relation (3) can be obtained from (7) by allowing n to equal t . Thus, as the call date approaches the maturity date or the reinvestment rate equals or exceeds the original investment rate, the callable security loses most of its uncertainty and becomes more like a fixed maturity bond.

As an illustration, consider the purchase of a 10-year \$500 par value 10 percent bond with semi-annual coupons for \$490. Exhibit 1 gives a comparison of yields and durations under various call assumptions. *Ceteris paribus* the further out the call privilege is from the purchase date the less impact callability has on duration.

⁷ Using an argument similar to the one made for (3), where $(Fr) \left\{ \frac{1}{i^2} \delta(1+i) + \frac{\delta'}{j^2} v_j^{t-n} (1+j) \right\} > C v_j^{t-n} (\delta + \delta')$,

$$\frac{1}{i} (n\delta) > \frac{1}{i}, \text{ and } \frac{1}{j} (t-n)\delta' > \frac{1}{j}.$$

Exhibit 1

Duration and Yield-Rate Comparisons

For a Callable Bond

	Duration (years)	Annual Yield Rate (percent)
With the Bond:		
A. Held to maturity	5.5447	10.35
B. Called at par after 5 years	3.4568	10.54
C. Called at par after 8 years	4.0537	10.40

Kellison [4] indicates that where a bond is called at par, it is possible to determine the effect calls have on yield rates. For a discounted bond, the earlier the call the higher the yield.⁸ The converse holds for a premium bond. Exhibit 1 bears this relation, for the discounted bond, the yield rates are higher for B and C than A. Unfortunately, there does not appear to be a similar rule for determining, a priori, changes in duration. If the bond considered here was purchased for \$535 and called in 5 years, the yield is 8.46 percent and the duration is 1.933. The same bond held to maturity has a yield of 8.97 percent and a duration of 5.514. A comparison of these findings with Exhibit 1 shows that whether the bond sold for a discount or premium, duration decreased as a result of a call after 5 years.

This result reflects the fact that duration will vary with redemption value, the time period before the call takes place, in addition to whether the bond originally sold for a premium or discount. What this implies for pension fund management is that for callable issues with several call dates and varying redemption values, duration should be calculated for each call date. A conservative policy with respect to duration and interest rate risk on income would be to assign the shortest possible duration to the callable security and plan on a call at that time. However, the fund can not be certain of cash flow from bond redemption until the maturity date of the callable security. Thus, uncertainty still remains in forecasting cash flows.

These findings provide a way to purchase callable issues and still acquire some of the potential benefits of duration. Bonds with call periods far out in the future and substantially higher yields than non-callable issues can give durations which approximate fixed maturity bonds. Since many corporations

⁸The reason is that the investor receives a gain on redemption earlier than if the security is held to maturity.

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prefer to offer only callable issues, purchase of such instruments according to this general guideline should allow the fund to diversify into corporate bonds without creating undue uncertainty with respect to duration.

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Annual Yield Rate (percent)

10.35

10.54

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